Bridging GCSE to A-Level

The Maths team would like to congratulate you on your GCSE achievements and welcome you to Newstead Wood.

We would like to also help you prepare for the next year and a half of your educational journey. It is very important that you attempt all these questions and bring what you could not do to class in September. It is your education and we are here to support you, but you need to take the initiative.

Some notes:

- You have chosen to do mathematics A-Level, so let us help you succeed.
- A-Level is fundamentally different from GCSE.
- The pace is faster, you are expected to do more independent work, and to seek support when you need it.
- Most importantly: cramming will not work. You need to understand why and how the maths work and apply it to new contexts. This is what these exercises are aimed to do. They will cover your GCSE topics with an added twist to aid deeper understanding.

We will cover:

1. Numbers
2. Algebra
3. Quadratic Functions and Equations
4. Solving Quadratic Simultaneous Equations
5. Graphs and Transformations
6. Area under the graph
7. Volume and surface areas

1 Numbers:

You have learnt the laws of indices. Here is a recap:

Remember surds are those roots you could not simplify, so $\sqrt{2}$ or $\sqrt{23}$ are surds, but $\sqrt{4} = 2$ and $\sqrt{25} = 5$ are not.

1. $x^a . x^b = x^{a+b}$, where $x, a,$ and b can be any number be it positive or negative integers, fractions, even surds or algebraic expressions.
2. $x^a \div x^b = \frac{x^a}{x^b} = x^{a-b}$
3. $(x^a)^b = x^{ab}$
4. $x^{\frac{a}{b}} = \left(\sqrt[b]{x}\right)^a$ – Think about what happens when b is even, can x be negative then? What if b is odd?
5. $x^1 = x$
6. $x^0 = 1$
7. $(xy)^a = x^a . y^a$. It is important for you to justify this with some numerical examples for yourself.

Examples:

$$16^{-\frac{3}{4}} = \frac{1}{16^{\frac{3}{4}}}$$

$$= \frac{1}{\left(\sqrt[4]{16}\right)^3}$$

$$= \frac{1}{2^3}$$

$$= \frac{1}{8}$$

$$(-9)^{-\frac{3}{2}} = \frac{1}{-9^{\frac{3}{2}}}$$

$$= \frac{1}{\sqrt{-9}}$$ Not a real number, so no answers exist!

1. Evaluate:

a) $121^{\frac{1}{2}}$

b) $8^{\frac{1}{3}}$

c) $216^{\frac{2}{3}}$

d) $25^{\frac{3}{2}}$

2. Simplify to a simple fraction or an integer:

a) $25^{-\frac{3}{2}}$

b) $(\frac{2}{3})^2$

c) $(\frac{1}{5})^{-1}$

d. $\left(-\frac{8}{27}\right)^{-\frac{2}{3}}$

e. $-4^{-\frac{3}{2}} + \frac{1}{2}\left(\frac{1}{8}\right)^{\frac{2}{3}}$

f. $-32^{-\frac{2}{5}} + \left(\frac{64}{125}\right)^{-\frac{1}{3}}$

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5},$ etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\frac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Example 1 Simplify $\sqrt{50}$

$\sqrt{50} = \sqrt{25 \times 2}$	**1** Choose two numbers that are factors of 50. One of the factors must be a square number
$= \sqrt{25} \times \sqrt{2}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 5 \times \sqrt{2}$	**3** Use $\sqrt{25} = 5$
$= 5\sqrt{2}$	

Example 2 Simplify $\sqrt{147} - 2\sqrt{12}$

$\sqrt{147} - 2\sqrt{12}$ $= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$	**1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number
$= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$	**3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$
$= 7\sqrt{3} - 4\sqrt{3}$ $= 3\sqrt{3}$	**4** Collect like terms

Example 3 Simplify $\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$

$\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$ $= \sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4}$	**1** Expand the brackets. A common mistake here is to write $\left(\sqrt{7}\right)^2 = 49$
$= 7 - 2$ $= 5$	**2** Collect like terms: $-\sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7}$ $= -\sqrt{7}\sqrt{2} + \sqrt{7}\sqrt{2} = 0$

Example 5 Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

$\frac{\sqrt{2}}{\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$	**1** Multiply the numerator and denominator by $\sqrt{12}$
$= \frac{\sqrt{2} \times \sqrt{4 \times 3}}{12}$	**2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number
$= \frac{2\sqrt{2}\sqrt{3}}{12}$	**3** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ **4** Use $\sqrt{4} = 2$
$= \frac{\sqrt{2}\sqrt{3}}{6}$	**5** Simplify the fraction: $\frac{2}{12}$ simplifies to $\frac{1}{6}$

Example 6 Rationalise and simplify $\frac{3}{2+\sqrt{5}}$

$\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$	**1** Multiply the numerator and denominator by $2-\sqrt{5}$
$= \frac{3\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}$	**2** Expand the brackets
$= \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$	**3** Simplify the fraction
$= \frac{6-3\sqrt{5}}{-1}$ $= 3\sqrt{5}-6$	**4** Divide the numerator by -1 Remember to change the sign of all terms when dividing by -1

3. Write in an index form

a) $\sqrt{7}$

b) $\sqrt[3]{2}$

c) $\sqrt[5]{5}$

d) $\sqrt[3]{3^2}$

e) $\dfrac{1}{\sqrt[3]{2^2}}$

4. Simplify

a $\sqrt{72}+\sqrt{162}$

b $\sqrt{45}-2\sqrt{5}$

c $\sqrt{50}-\sqrt{8}$

d $\sqrt{75}-\sqrt{48}$

e $2\sqrt{28}+\sqrt{28}$

f $2\sqrt{12}-\sqrt{12}+\sqrt{27}$

5. Expand and simplify

a $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$

b $(3+\sqrt{3})(5-\sqrt{12})$

c $(4-\sqrt{5})(\sqrt{45}+2)$

d $(5+\sqrt{2})(6-\sqrt{8})$

6. Rationalise the denominator and simplify.

a $\dfrac{1}{3-\sqrt{5}}$

b $\dfrac{2}{4+\sqrt{3}}$

c $\dfrac{6}{5-\sqrt{2}}$

7. Expand and simplify

$$\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)$$

8. Rationalise the denominator and simplify where possible.

a $\dfrac{1}{\sqrt{9}-\sqrt{8}}$

b $\dfrac{1}{\sqrt{x}-\sqrt{y}}$

2 Algebra:

Remember that algebraic expressions are just generalisation of arithmetic with numbers. All the rules that applied to numbers will apply here. The letters are just placeholders for numbers. We will now apply the above index laws to algebraic expressions.

When it comes to changing the sunject of a formula, just think of it as solving for that variable and follow the same rules as solving equations.

Examples:

Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$

Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	**1** Use the rule $a^m \times a^n = a^{m+n}$ **2** Use the rule $\frac{a^m}{a^n} = a^{m-n}$

Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged

Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ **2** Use the rule $\frac{1}{a^m} = a^{-m}$

$$(27x^6)^{-\frac{4}{3}}\left(3x^{\frac{4}{3}}\right) = \frac{1}{(27x^6)^{\frac{4}{3}}}\left(3x^{\frac{4}{3}}\right) = \frac{3x^{\frac{4}{3}}}{27^{\frac{4}{3}}(x^6)^{\frac{4}{3}}}$$

$$= \frac{3x^{\frac{4}{3}}}{\left(\sqrt[3]{27}\right)^4 x^8}$$

$$= \frac{3x^{\frac{4}{3}-8}}{(3)^4}$$

$$= \frac{3x^{-\frac{20}{3}}}{81}$$

$$= \frac{1}{27x^{\frac{20}{3}}}$$

Make t the subject of the formula $r = 2t - \pi t$.

$r = 2t - \pi t$ $r = t(2 - \pi)$ $t = \frac{r}{2 - \pi}$	**1** All the terms containing t are already on one side and everything else is on the other side. **2** Factorise as t is a common factor. **3** Divide throughout by $2 - \pi$.

Make t the subject of the formula $\frac{t+r}{5} = \frac{3t}{2}$.

$\frac{t+r}{5} = \frac{3t}{2}$ $2t + 2r = 15t$ $2r = 13t$ $t = \frac{2r}{13}$	**1** Remove the fractions first by multiplying throughout by 10. **2** Get the terms containing t on one side and everything else on the other side and simplify. **3** Divide throughout by 13.

Make t the subject of the formula $r = \frac{3t+5}{t-1}$.

$r = \frac{3t+5}{t-1}$ $r(t - 1) = 3t + 5$ $rt - r = 3t + 5$ $rt - 3t = 5 + r$ $t(r - 3) = 5 + r$ $t = \frac{5+r}{r-3}$	**1** Remove the fraction first by multiplying throughout by $t - 1$. **2** Expand the brackets. **3** Get the terms containing t on one side and everything else on the other side. **4** Factorise the LHS as t is a common factor. **5** Divide throughout by $r - 3$.

1. Simplify

a	$\dfrac{3x^2 \times x^3}{2x^2}$	**b**	$\dfrac{10x^5}{2x^2 \times x}$
c	$\dfrac{3x \times 2x^3}{2x^3}$	**d**	$\dfrac{7x^3y^2}{14x^5y}$
e	$\dfrac{y^2}{y^{\frac{1}{2}} \times y}$	**f**	$\dfrac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$
g	$\dfrac{\left(2x^2\right)^3}{4x^0}$	**h**	$\dfrac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

2. Write as sums of powers of x.

a $\dfrac{x^5+1}{x^2}$ **b** $x^2\left(x+\dfrac{1}{x}\right)$ **c** $x^{-4}\left(x^2+\dfrac{1}{x^3}\right)$

3. Simplify

a. $(27x)^{\frac{2}{3}}\left(3x^{\frac{4}{5}}\right)$ b. $\left(-2x^{\frac{3}{2}}\right)\,(64x^6)^{-\frac{4}{3}}$ c. $-\left(\frac{x^3}{64}\right)^{-\frac{4}{3}}\left(\frac{4^{-\frac{3}{2}}}{x^8}\right)$

d. $\left(\frac{x^{12}}{81}\right)^{\frac{3}{4}}\left(-\frac{x^9}{27}\right)^{-\frac{2}{3}}\left(\frac{1}{4}x^{-\frac{4}{5}}\right)$

Factorising quadratics, especially non-monial quadratics is something you should remembre well from GCSE. Here are a couple of exampes as reminder:

$3x^2+10x+8$

$= 3x^2+6x+4x+8$

$= 3x(x+2)+4(x+2)$

$= (3x+4)(x+2)$

We needed p and q such that

$p+q=10$ *and* $p \times q = 3 \times 8 = 24$

$-5x^4y+20x^3y^2-20x^2y^3$

$= -5x^2y(x^2-4xy+4y^2)$

$= -5x^2y(x-2y)^2$

4.

a) $x^2+4xy-12y^2$ c) $6x^2+5x+1$

b) $-x^2-2x+35$ d) $12x^3-12x^2-9x$

5. Factorise $9b^2x+9b^2y-16x-16y$

6. Factorise $x^2-8x+16-y^2$

7. Factorise fully:

a. $75x^4 - 48y^4$

b. $b^2 - 10b + 25 - a^2$

c. $4x^4y - 40x^3y + 100x^2y$

We can factorise powers that are not integers as well. Here is an example:

$8x^{\frac{7}{3}} - 4x^{\frac{11}{3}} + 12x^{\frac{5}{3}}$ =

Write as $4x^{\frac{5}{3}}(\quad\quad)$ and fill whatever is left to get $\boxed{4x^{\frac{5}{3}}(2x^{\frac{2}{3}} - 4x^2 + 3)}$

8. Factor $10x^{\frac{3}{8}} - 15x^{\frac{1}{3}} + 20x$

9. Simplify

$x^{\frac{3}{2}} - x^{\frac{1}{2}}$

$4x^{\frac{-2}{3}} - 8x^{\frac{1}{3}}$

$(x-5)^{-\frac{1}{2}} - (x-5)^{-\frac{3}{2}}$

$12x^{\frac{-3}{4}} - 8x^{\frac{1}{4}}$

$5(4x+3)^{-1} - 4(5x+1)(4x+3)^{-2}$

$-\frac{1}{2}(3x)(1-x^2)^{-\frac{3}{2}}(-2x) + 3(1-x^2)^{-\frac{1}{2}}$

10.

Expand and simplify.

a $\left(x+\frac{1}{x}\right)\left(x-\frac{2}{x}\right)$ **b** $\left(x+\frac{1}{x}\right)^2$

Change the subject of each formula to the letter given in brackets.

11 $e(9 + x) = 2e + 1$ [e] **12** $y = \frac{2x+3}{4-x}$ [x]

13 Make r the subject of the following formulae.

a $A = \pi r^2$ **b** $V = \frac{4}{3}\pi r^3$ **c** $P = \pi r + 2r$ **d** $V = \frac{2}{3}\pi r^2 h$

14 Make x the subject of the following formulae.

a $\frac{xy}{z} = \frac{ab}{cd}$ **b** $\frac{4\pi cx}{d} = \frac{3z}{py^2}$

15 Make $\sin B$ the subject of the formula $\frac{a}{\sin A} = \frac{b}{\sin B}$

16 Make $\cos B$ the subject of the formula $b^2 = a^2 + c^2 - 2ac\cos B$.

17 Make x the subject of the following equations.

a $\frac{p}{q}(sx+t) = x-1$

b $\frac{p}{q}(ax+2y) = \frac{3p}{q^2}(x-y)$

3 Quadratic Functions & Equations:

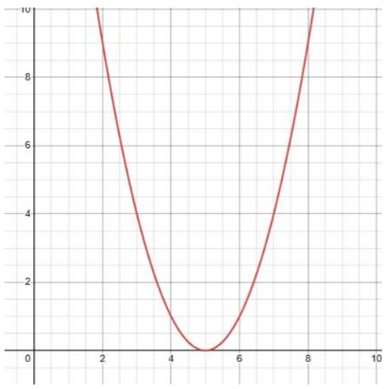
A quadratic function is given by

$$f(x) = ax^2 + bx + c, \quad a \neq 0$$

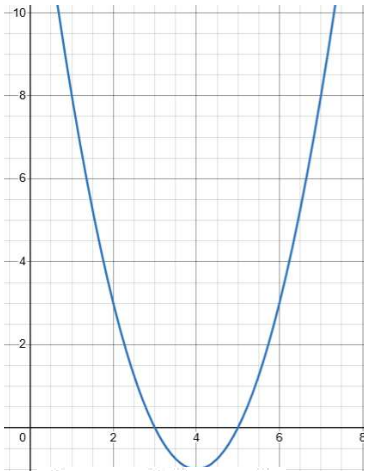
When you insert a number (value) for x, You will receive a value for $f(x)$.

A quadratic equation is when we are interested in a specific value of $f(x)$. We usually then solve it to find the corresponding valuefor x. The most important one is when $f(x) = 0$, which means where the graph of the function crosses the x-axis.

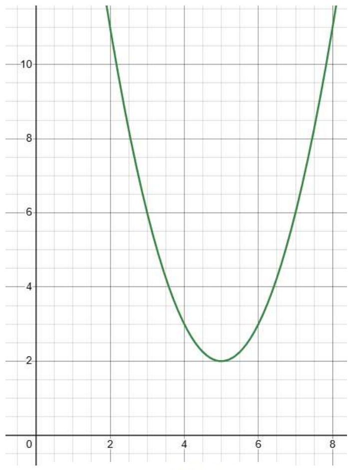
As you can see below not all quadratic functions will cross the x-axis. Try and think why is that before we go through this in details.



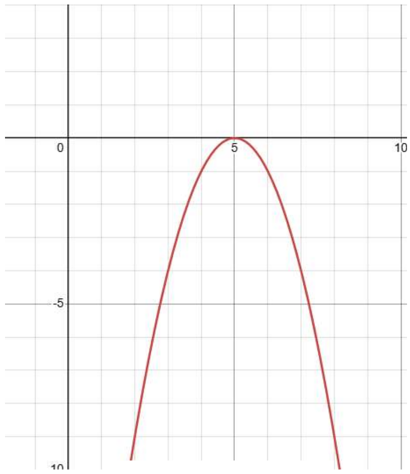
$(x-5)^2$



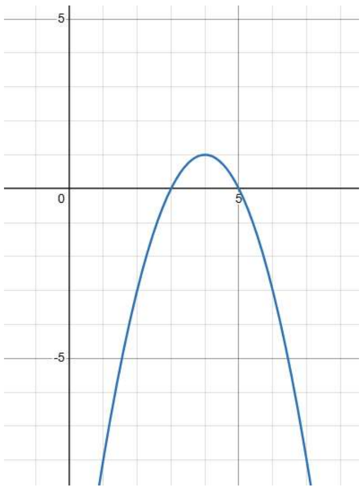
$(x-5)(x-3)$



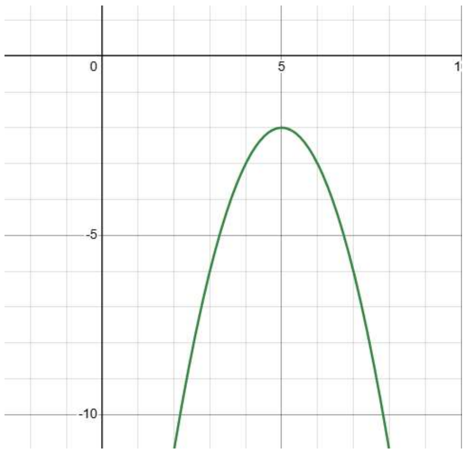
$(x-5)^2+2$



$-(x-5)^2$



$-(x-5)(x-3)$



$-(x-5)^2-2$

They key to how many roots you may have lies in your quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Think what happens if

1. $b^2 - 4ac = 0$? Then whatever comes after $\pm$ is 0. So you have one answer only. We call this a repeated root.

2. $b^2 - 4ac < 0$? Then the square root will have no answers. So, there are no real roots.

3. $b^2 - 4ac > 0$? You will get two distinct real roots.

Example:

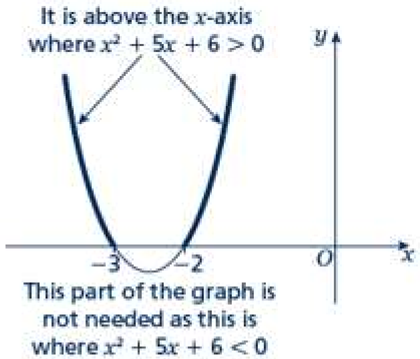
Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	**2** Substitute $a = 3, b = -7, c = -2$ into the formula.
$x = \frac{7 \pm \sqrt{73}}{6}$	**3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.
So $x = \frac{7 - \sqrt{73}}{6}$ or $x = \frac{7 + \sqrt{73}}{6}$	**4** Write down both the solutions.

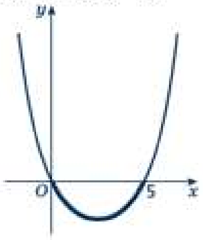
You can also use this information to see for which values of x your graph is above or below 0, thus answering quadratic inequality questions.

Examples:

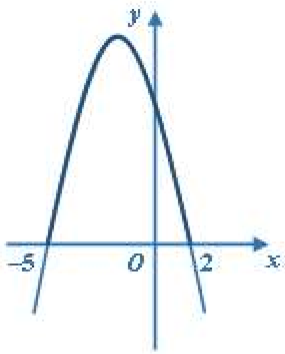
Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$

$x^2 + 5x + 6 = 0$ $(x + 3)(x + 2) = 0$ $x = -3$ or $x = -2$	**1** Solve the quadratic equation by factorising.
	**2** Sketch the graph of $y = (x + 3)(x + 2)$ **3** Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$
$x < -3$ or $x > -2$	**4** Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$

Find the set of values of x which satisfy $x^2 - 5x \leq 0$

$x^2 - 5x = 0$ $x(x - 5) = 0$ $x = 0$ or $x = 5$  $0 \leq x \leq 5$	**1** Solve the quadratic equation by factorising. **2** Sketch the graph of $y = x(x - 5)$ **3** Identify on the graph where $x^2 - 5x \leq 0$, i.e. where $y \leq 0$ **4** Write down the values which satisfy the inequality $x^2 - 5x \leq 0$

Find the set of values of x which satisfy $-x^2 - 3x + 10 \geq 0$

$-x^2 - 3x + 10 = 0$ $(-x + 2)(x + 5) = 0$ $x = 2$ or $x = -5$  $-5 \leq x \leq 2$	**1** Solve the quadratic equation by factorising. **2** Sketch the graph of $y = (-x + 2)(x + 5) = 0$ **3** Identify on the graph where $-x^2 - 3x + 10 \geq 0$, i.e. where $y \geq 0$ **3** Write down the values which satisfy the inequality $-x^2 - 3x + 10 \geq 0$

Of course you can use factorising (where possible) and completing the square to find the roots. You can also use comleting the square to find the turning point of the graph of a quadratic function.

- For the quadratic function $f(x) = a(x + p)^2 + q$, the graph of $y = f(x)$ has a turning point at $(-p, q)$

Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$ $2\left(x^2 - \frac{7}{2}x\right) + 4 = 0$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$
$2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] + 4 = 0$	**2** Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$
$2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8} + 4 = 0$	**3** Expand the square brackets.
$2\left(x - \frac{7}{4}\right)^2 - \frac{17}{8} = 0$	**4** Simplify.
$2\left(x - \frac{7}{4}\right)^2 = \frac{17}{8}$	**5** Rearrange the equation to work out x. First, add $\frac{17}{8}$ to both sides.
$\left(x - \frac{7}{4}\right)^2 = \frac{17}{16}$	**6** Divide both sides by 2.
$x - \frac{7}{4} = \pm\frac{\sqrt{17}}{4}$	**7** Square root both sides. Remember that the square root of a value gives two answers.
$x = \pm\frac{\sqrt{17}}{4} + \frac{7}{4}$	**8** Add $\frac{7}{4}$ to both sides.
So $x = \frac{7}{4} - \frac{\sqrt{17}}{4}$ or $x = \frac{7}{4} + \frac{\sqrt{17}}{4}$	**9** Write down both the solutions.

1. Solve, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

2. Find the nature(i.e. the numer of real) roots of the following quadratics

$x^2 + 6x + 9 = 0$

$x^2 + 9x + 20 = 0$

$2x^2 - 10x + 8 = 0$

$x^2 + 5x + 10 = 0$

$x^2 + 6x + 3 = 0$

$2x^2 + 6x + 4 = 0$

$3x^2 - 5x = -4$

$9x^2 - 6x = -9$

$10x^2 - 4x = 8$

$3x^2 - 2x - 5 = 0$

3 Find the set of values of x for which $2x^2 - 7x + 3 < 0$

4 Find the set of values of x for which $4x^2 + 4x - 3 > 0$

5 Find the set of values of x for which $12 + x - x^2 \geq 0$

Find the set of values which satisfy the following inequalities.

6 $x^2 + x \leq 6$

7 $x(2x - 9) < -10$

8 $6x^2 \geq 15 + x$

9. Solve by completing the square.

a $(x - 4)(x + 2) = 5$

b $2x^2 + 6x - 7 = 0$

c $x^2 - 5x + 3 = 0$

4 Solving Quadratic Simultaneous Equations:

A set of simultaneaous equations where one is a quadratic and the other a line looks for the crossing point of the line and the curve (or circle in some cases).

Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Example:

Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x = \frac{5-3y}{2}$	**1** Rearrange the first equation.
$2y^2 + \left(\frac{5-3y}{2}\right)y = 12$	**2** Substitute $\frac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y.
$2y^2 + \frac{5y-3y^2}{2} = 12$ $4y^2 + 5y - 3y^2 = 24$ $y^2 + 5y - 24 = 0$	**3** Expand the brackets and simplify.
$(y + 8)(y - 3) = 0$	**4** Factorise the quadratic equation.
So $y = -8$ or $y = 3$	**5** Work out the values of y.
Using $2x + 3y = 5$ When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$ When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$ So the solutions are $x = 14.5$, $y = -8$ and $x = -2$, $y = 3$	**6** To find the value of x, substitute both values of y into one of the original equations.
Check: equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES and $2 \times (-2) + 3 \times 3 = 5$ YES equation 2: $2\times(-8)^2 + 14.5\times(-8) = 12$ YES and $2 \times (3)^2 + (-2) \times 3 = 12$ YES	**7** Substitute both pairs of values of x and y into both equations to check your answers.

Solve these simultaneous equations.

1 $y = 2x + 1$
$x^2 + y^2 = 10$

2 $y = 6 - x$
$x^2 + y^2 = 20$

3 $y = x - 3$
$x^2 + y^2 = 5$

4 $y = 9 - 2x$
$x^2 + y^2 = 17$

5 $y = 3x - 5$
$y = x^2 - 2x + 1$

6 $y = x - 5$
$y = x^2 - 5x - 1$

7 $y = x + 5$
$x^2 + y^2 = 25$

8 $y = 2x - 1$
$x^2 + xy = 24$

9 $y = 2x$
$y^2 - xy = 8$

10 $2x + y = 11$
$xy = 15$

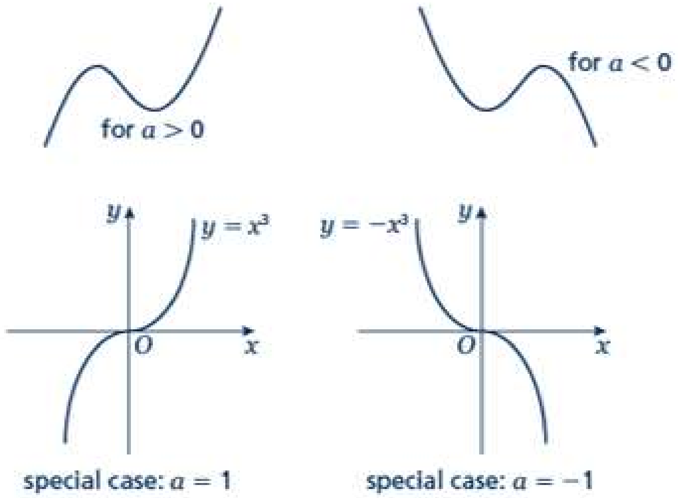
11 $x - y = 1$
$x^2 + y^2 = 3$

12 $y - x = 2$
$x^2 + xy = 3$

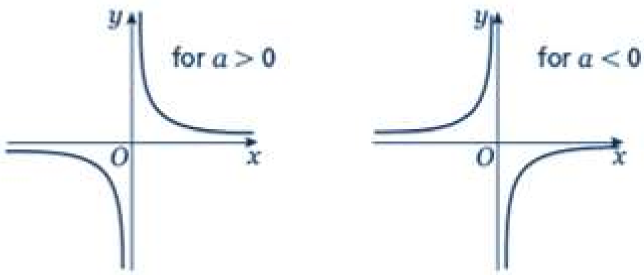
5 Graphs and Transformations:

Key points

- The graph of a cubic function, which can be written in the form $y = ax^3 + bx^2 + cx + d$, where $a \neq 0$, has one of the shapes shown here.



- The graph of a reciprocal function of the form $y = \frac{a}{x}$ has one of the shapes shown here.



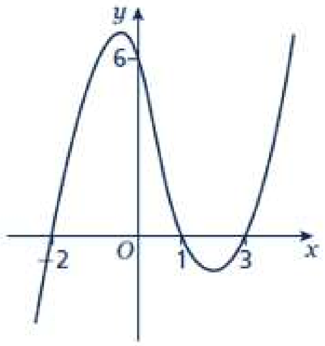
- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- Where appropriate, mark and label the asymptotes on the graph.
- Asymptotes are lines (usually horizontal or vertical) which the curve gets closer to but never touches or crosses. Asymptotes usually occur with reciprocal functions. For example, the asymptotes for the graph of $y = \frac{a}{x}$ are the two axes (the lines $y = 0$ and $x = 0$).
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- A double root is when two of the solutions are equal. For example $(x - 3)^2(x + 2)$ has a double root at $x = 3$.
- When there is a double root, this is one of the turning points of a cubic function.

Remember that "drawing" is accurate, while "sketch" gives the general idea about how a function behaves.

Examples:

Sketch the graph of $y = (x - 3)(x - 1)(x + 2)$

<table>
<tr><td colspan="2">To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape.</td></tr>
<tr><td>When $x = 0$, $y = (0 - 3)(0 - 1)(0 + 2)$
$= (-3) \times (-1) \times 2 = 6$
The graph intersects the y-axis at (0, 6)

When $y = 0$, $(x - 3)(x - 1)(x + 2) = 0$
So $x = 3$, $x = 1$ or $x = -2$
The graph intersects the x-axis at
(−2, 0), (1, 0) and (3, 0)

</td>
<td>1 Find where the graph intersects the axes by substituting $x = 0$ and $y = 0$. Make sure you get the coordinates the right way around, (x, y).
2 Solve the equation by solving $x - 3 = 0$, $x - 1 = 0$ and $x + 2 = 0$

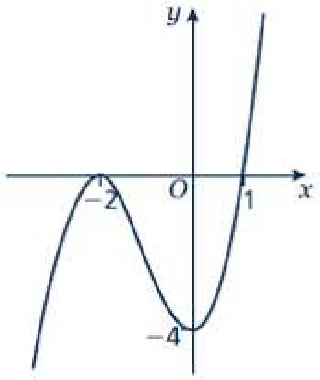
3 Sketch the graph.
$a = 1 > 0$ so the graph has the shape:

</td></tr>
</table>

Sketch the graph of $y = (x + 2)^2(x - 1)$

<table>
<tr><td colspan="2">To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape.</td></tr>
<tr><td>When $x = 0$, $y = (0 + 2)^2(0 - 1)$
$= 2^2 \times (-1) = -4$
The graph intersects the y-axis at (0, −4)

When $y = 0$, $(x + 2)^2(x - 1) = 0$
So $x = -2$ or $x = 1$

(−2, 0) is a turning point as $x = -2$ is a double root.
The graph crosses the x-axis at (1, 0)

</td>
<td>1 Find where the graph intersects the axes by substituting $x = 0$ and $y = 0$.

2 Solve the equation by solving $x + 2 = 0$ and $x - 1 = 0$

3 $a = 1 > 0$ so the graph has the shape:

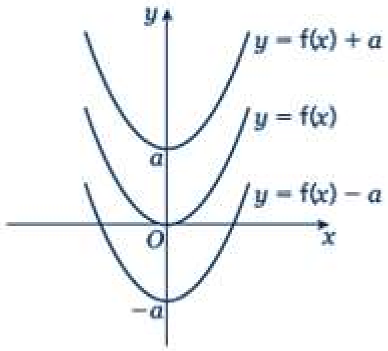
</td></tr>
</table>

For transformation of graphs remember:

Key points

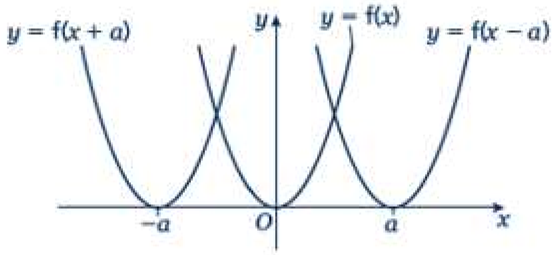
- The transformation $y = f(x) \pm a$ is a translation of $y = f(x)$ parallel to the y-axis; it is a vertical translation.

  As shown on the graph,
  - $y = f(x) + a$ translates $y = f(x)$ up
  - $y = f(x) - a$ translates $y = f(x)$ down.

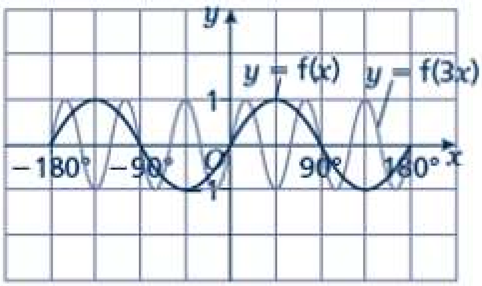


- The transformation $y = f(x \pm a)$ is a translation of $y = f(x)$ parallel to the x-axis; it is a horizontal translation.

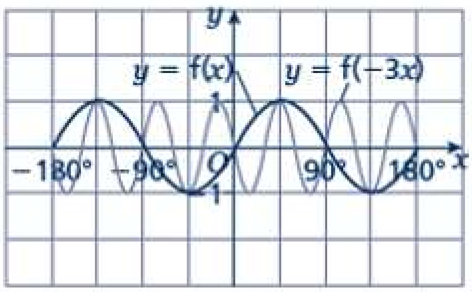
  As shown on the graph,
  - $y = f(x + a)$ translates $y = f(x)$ to the left
  - $y = f(x - a)$ translates $y = f(x)$ to the right.



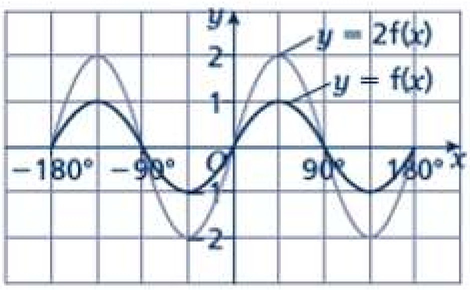
- The transformation $y = f(ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ parallel to the x-axis.



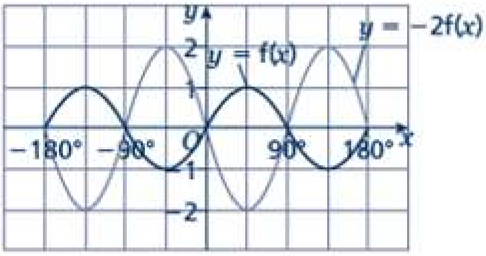
- The transformation $y = f(-ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ parallel to the x-axis and then a reflection in the y-axis.



- The transformation $y = af(x)$ is a vertical stretch of $y = f(x)$ with scale factor a parallel to the y-axis.



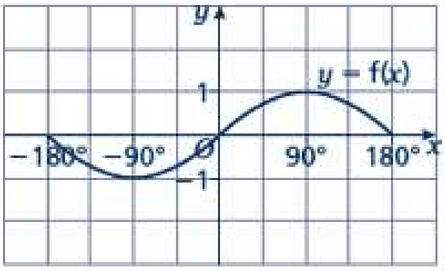
- The transformation $y = -a\text{f}(x)$ is a vertical stretch of $y = \text{f}(x)$ with scale factor a parallel to the y-axis and then a reflection in the x-axis.

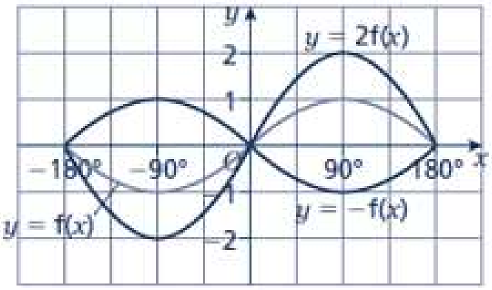


Examples:

The graph shows the function $y = \text{f}(x)$.

Sketch and label the graphs of $y = 2\text{f}(x)$ and $y = -\text{f}(x)$.



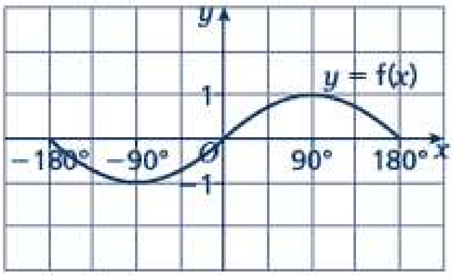


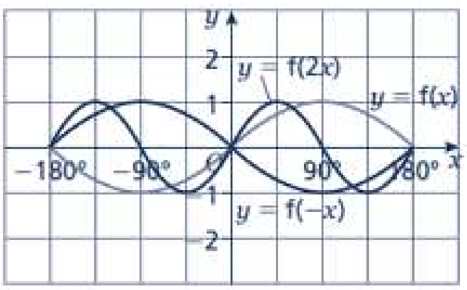
The function $y = 2\text{f}(x)$ is a vertical stretch of $y = \text{f}(x)$ with scale factor 2 parallel to the y-axis.

The function $y = -\text{f}(x)$ is a reflection of $y = \text{f}(x)$ in the x-axis.

The graph shows the function $y = \text{f}(x)$.

Sketch and label the graphs of $y = \text{f}(2x)$ and $y = \text{f}(-x)$.





The function $y = \text{f}(2x)$ is a horizontal stretch of $y = \text{f}(x)$ with scale factor $\frac{1}{2}$ parallel to the x-axis.

The function $y = \text{f}(-x)$ is a reflection of $y = \text{f}(x)$ in the y-axis.

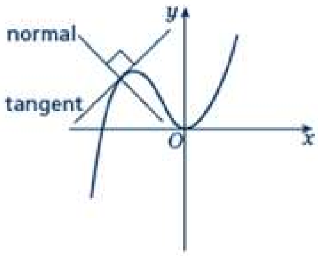
1 Here are six equations.

A	$y = \frac{5}{x}$	**B**	$y = x^2 + 3x - 10$	**C**	$y = x^3 + 3x^2$
D	$y = 1 - 3x^2 - x^3$	**E**	$y = x^3 - 3x^2 - 1$	**F**	$x + y = 5$

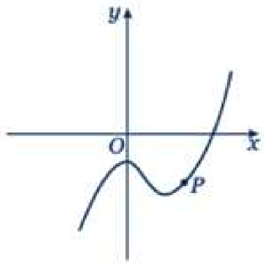
> **Hint**
>
> Find where each of the cubic equations cross the y-axis.

Here are six graphs.

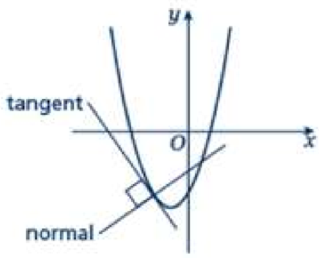
i



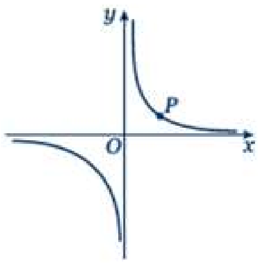
ii



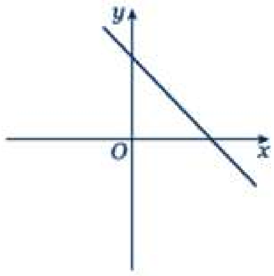
iii



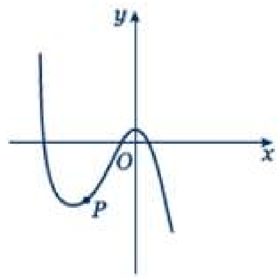
iv



v



vi



a Match each graph to its equation.

b Copy the graphs ii, iv and vi and draw the tangent and normal each at point P.

Sketch the following graphs

2 $y = 2x^3$

3 $y = x(x - 2)(x + 2)$

4 $y = (x + 1)(x + 4)(x - 3)$

5 $y = (x + 1)(x - 2)(1 - x)$

6 $y = (x - 3)^2(x + 1)$

7 $y = (x - 1)^2(x - 2)$

8 $y = \frac{3}{x}$

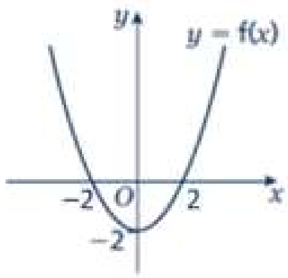
> **Hint:** Look at the shape of $y = \frac{a}{x}$ in the second key point.

9 $y = -\frac{2}{x}$

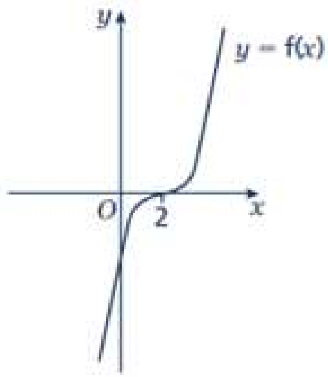
10 Sketch the graph of $y = \dfrac{1}{x+2}$

11 Sketch the graph of $y = \dfrac{1}{x-1}$

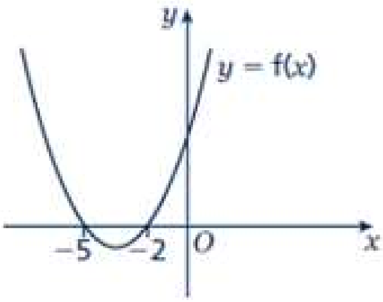
12 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch and label the graphs of $y = f(x) + 4$ and $y = f(x + 2)$.



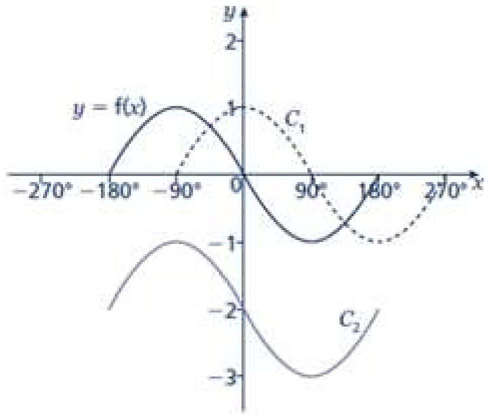
13 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch and label the graphs of $y = f(x + 3)$ and $y = f(x) - 3$.



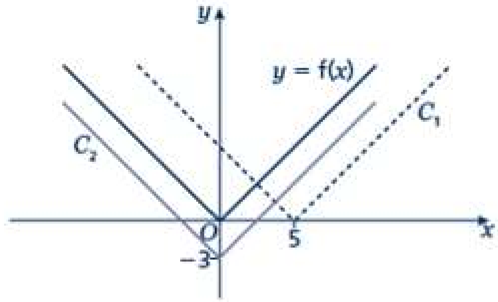
14 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch the graph of $y = f(x - 5)$.



15 The graph shows the function $y = f(x)$ and two transformations of $y = f(x)$, labelled C_1 and C_2. Write down the equations of the translated curves C_1 and C_2 in function form.



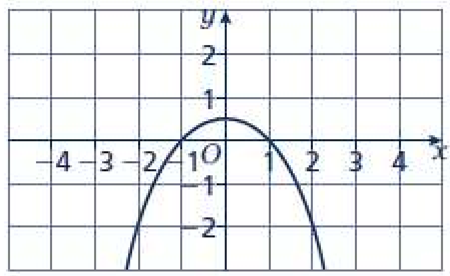
16 The graph shows the function $y = f(x)$ and two transformations of $y = f(x)$, labelled C_1 and C_2. Write down the equations of the translated curves C_1 and C_2 in function form.



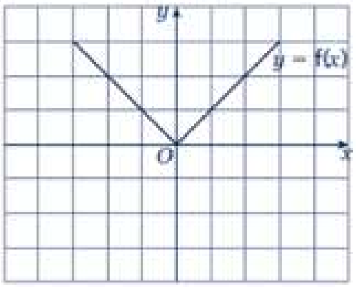
17 The graph shows the function $y = f(x)$.

a Sketch the graph of $y = f(x) + 2$

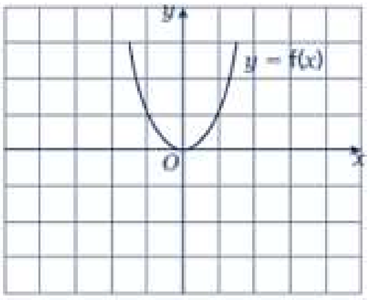
b Sketch the graph of $y = f(x + 2)$



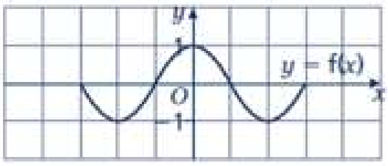
18 The graph shows the function $y = \mathrm{f}(x)$. Copy the graph and on the same axes sketch and label the graphs of $y = -2\mathrm{f}(x)$ and $y = \mathrm{f}(3x)$.



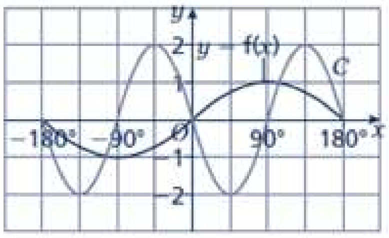
19 The graph shows the function $y = \mathrm{f}(x)$. Copy the graph and, on the same axes, sketch and label the graphs of $y = -\mathrm{f}(x)$ and $y = \mathrm{f}\left(\frac{1}{2}x\right)$.



20 The graph shows the function $y = \mathrm{f}(x)$. Copy the graph and, on the same axes, sketch the graph of $y = -\mathrm{f}(2x)$.



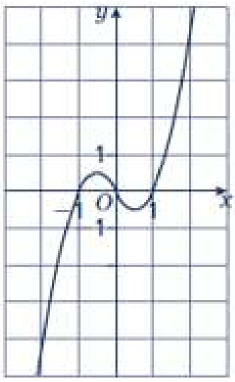
21 The graph shows the function $y = \mathrm{f}(x)$ and a transformation labelled C. Write down the equation of the translated curve C in function form.



22 The graph shows the function $y = \mathrm{f}(x)$.

a Sketch the graph of $y = -\mathrm{f}(x)$.

b Sketch the graph of $y = 2\mathrm{f}(x)$.



23

a Sketch and label the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (x - 1)(x + 1)$.

b On the same axes, sketch and label the graphs of $y = \mathrm{f}(x) - 2$ and $y = \mathrm{f}(x + 2)$.

24

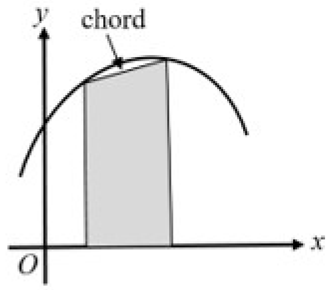
a Sketch and label the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = -(x + 1)(x - 2)$.

b On the same axes, sketch and label the graph of $y = \mathrm{f}\left(-\frac{1}{2}x\right)$.

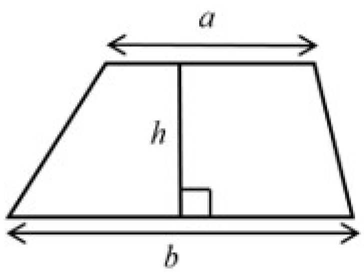
6 Area Under Graphs:

Key points

- To estimate the area under a curve, draw a chord between the two points you are finding the area between and straight lines down to the horizontal axis to create a trapezium. The area of the trapezium is an approximation for the area under a curve.

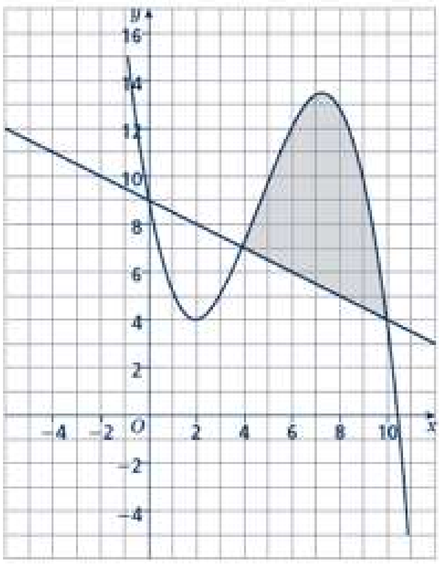


- The area of a trapezium = $\frac{1}{2}h(a+b)$



Example:

Estimate the shaded area.
Use three strips of width 2 units.



1 Use a table to record y on the curve for each value of x.

x	4	6	8	10
y	7	12	13	4

2 Use a table to record y on the straight line for each value of x.

x	4	6	8	10
y	7	6	5	4

3 Work out the dimensions of each trapezium. The distances between the y-values on the curve and the y-values on the straight line give the values for a.

Trapezium 1:
$a_1 = 7 - 7 = 0$, $b_1 = 12 - 6 = 6$
Trapezium 2:
$a_2 = 12 - 6 = 6$, $b_2 = 13 - 5 = 8$
Trapezium 3:
$a_3 = 13 - 5 = 8$, $a_3 = 4 - 4 = 0$

4 Work out the area of each trapezium. $h = 2$ since the width of each trapezium is 2 units.

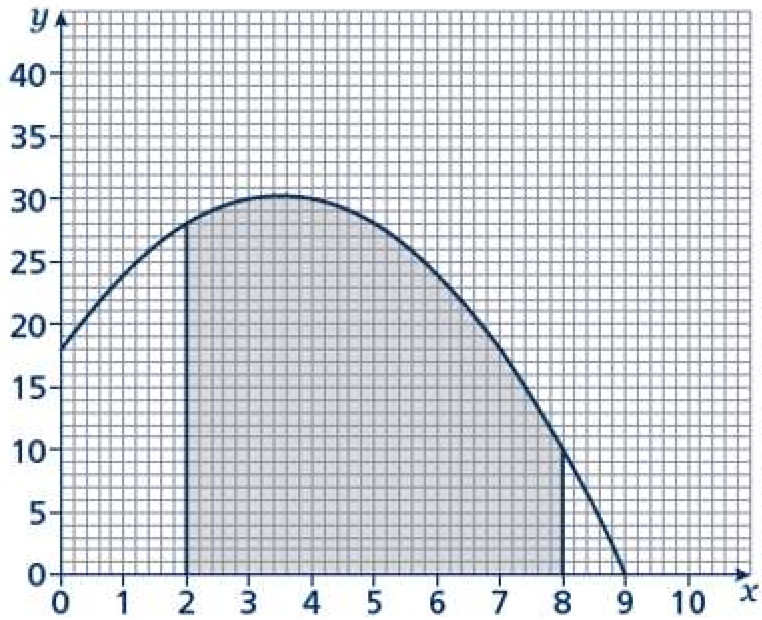
$$\frac{1}{2}h(a_1 + b_1) = \frac{1}{2} \times 2(0 + 6) = 6$$
$$\frac{1}{2}h(a_2 + b_2) = \frac{1}{2} \times 2(6 + 8) = 14$$
$$\frac{1}{2}h(a_3 + b_3) = \frac{1}{2} \times 2(8 + 0) = 8$$

5 Work out the total area. Remember to give units with your answer.

Area = 6 + 14 + 8 = 28 units^2

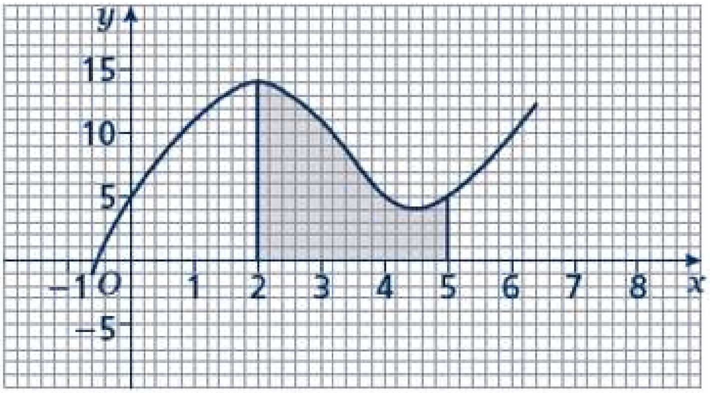
1 Estimate the area of the region between the curve $y = (5 - x)(x + 2)$ and the x-axis from $x = 1$ to $x = 5$.
Use four strips of width 1 unit.

2 Estimate the shaded area shown on the axes.
Use six strips of width 1 unit.

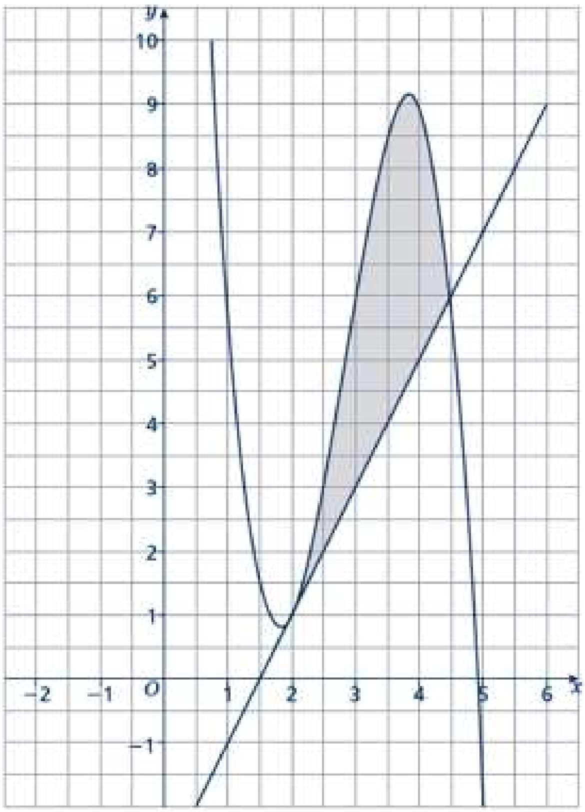


3 Estimate the area of the region between the curve $y = x^2 - 8x + 18$ and the x-axis from $x = 2$ to $x = 6$.
Use four strips of width 1 unit.

4 Estimate the shaded area.
Use six strips of width $\frac{1}{2}$ unit.

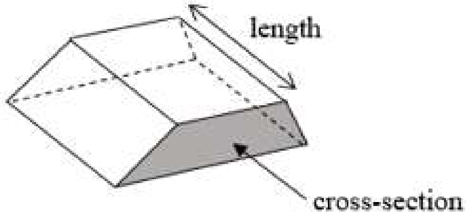


5 Estimate the shaded area using five strips of equal width.

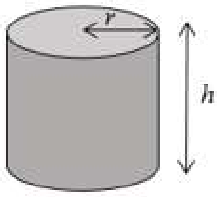


7 Volumes and Surface Areas:

Key points

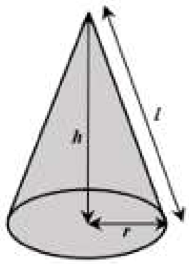


- Volume of a prism = cross-sectional area × length.
- The surface area of a 3D shape is the total area of all its faces.
- Volume of a pyramid = $\frac{1}{3}$ × area of base × vertical height.



- Volume of a cylinder = $\pi r^2 h$
- Total surface area of a cylinder = $2\pi r^2 + 2\pi rh$

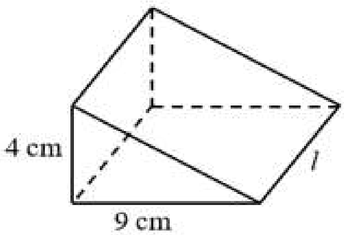
- Volume of a sphere = $\frac{4}{3}\pi r^3$
- Surface area of a sphere = $4\pi r^2$



- Volume of a cone = $\frac{1}{3}\pi r^2 h$
- Total surface area of a cone = $\pi rl + \pi r^2$

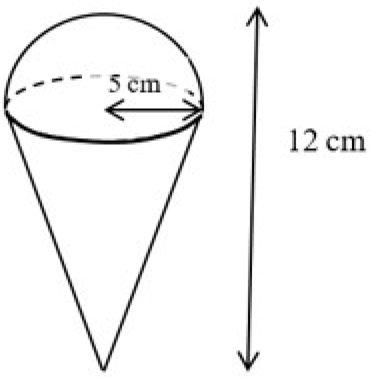
Examples:

The triangular prism has volume 504 cm³.
Work out its length.



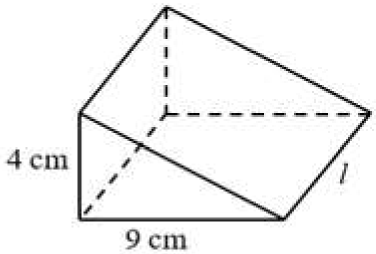
$V = \frac{1}{2}bhl$	**1** Write out the formula for the volume of a triangular prism.
$504 = \frac{1}{2} \times 9 \times 4 \times l$	**2** Substitute known values into the formula.
$504 = 18 \times l$	**3** Simplify
$l = 504 \div 18$	**4** Rearrange to work out l.
$= 28$ cm	**5** Remember the units.

Calculate the volume of the 3D solid.
Give your answer in terms of π.



Total volume = volume of hemisphere + Volume of cone $= \frac{1}{2}$ of $\frac{4}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$	**1** The solid is made up of a hemisphere radius 5 cm and a cone with radius 5 cm and height $12 - 5 = 7$ cm.
Total volume $= \frac{1}{2} \times \frac{4}{3} \times \pi \times 5^3 + \frac{1}{3} \times \pi \times 5^2 \times 7$	**2** Substitute the measurements into the formula for the total volume.
$= \frac{425}{3}\pi$ cm^3	**3** Remember the units.

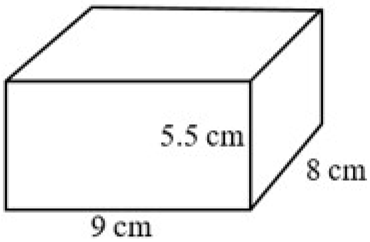
The triangular prism has volume 504 cm^3.
Work out its length.



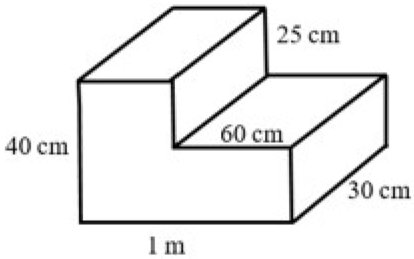
$V = \frac{1}{2} bhl$	**1** Write out the formula for the volume of a triangular prism.
$504 = \frac{1}{2} \times 9 \times 4 \times l$	**2** Substitute known values into the formula.
$504 = 18 \times l$	**3** Simplify
$l = 504 \div 18$ $= 28$ cm	**4** Rearrange to work out l. **5** Remember the units.

1 Work out the volume of each solid.
Leave your answers in terms of π where appropriate.

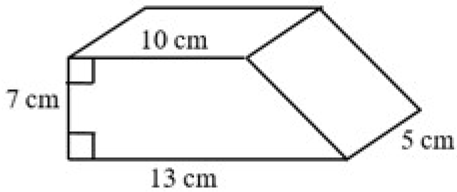
a



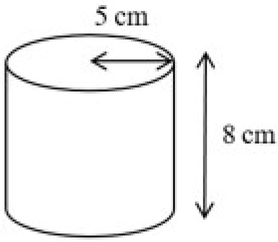
b



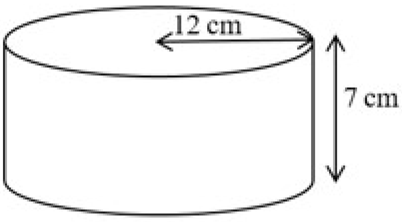
c



d



e

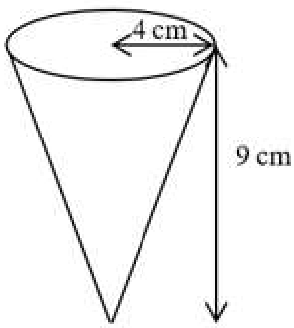


f a sphere with radius 7 cm

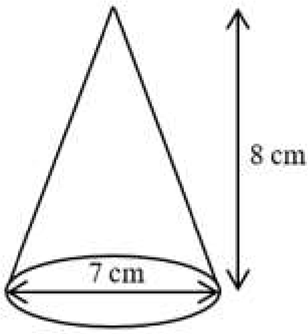
g a sphere with diameter 9 cm

h a hemisphere with radius 3 cm

i

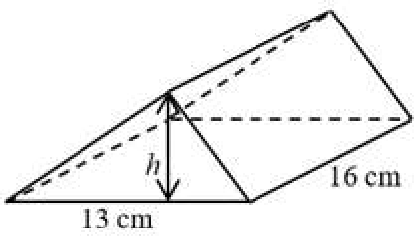


j

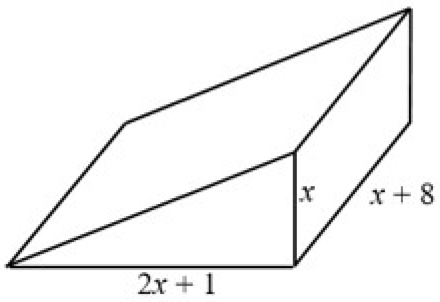


2 A cuboid has width 9.5 cm, height 8 cm and volume 1292 cm^3.
Work out its length.

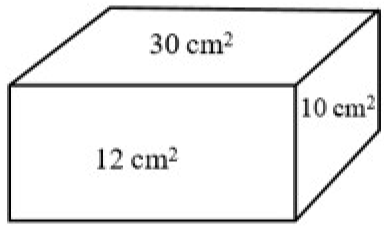
3 The triangular prism has volume 1768 cm^3.
Work out its height.



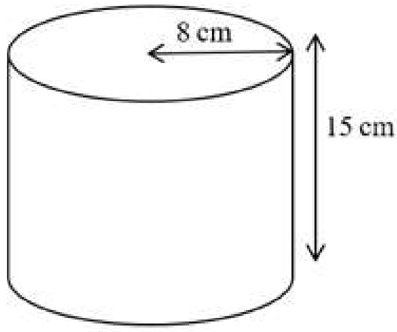
4 The diagram shows a solid triangular prism.
All the measurements are in centimetres.
The volume of the prism is V cm³.
Find a formula for V in terms of x.
Give your answer in simplified form.



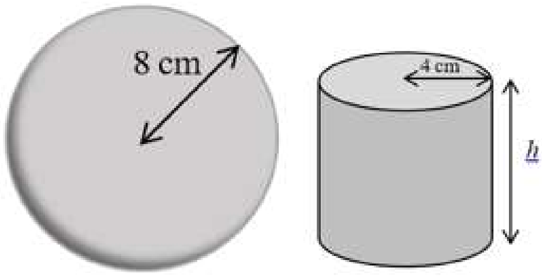
5 The diagram shows the area of each of three faces of a cuboid.
The length of each edge of the cuboid is a whole number of centimetres.
Work out the volume of the cuboid.



6 The diagram shows a large catering size tin of beans in the shape of a cylinder.
The tin has a radius of 8 cm and a height of 15 cm.
A company wants to make a new size of tin.
The new tin will have a radius of 6.7 cm.
It will have the same volume as the large tin.
Calculate the height of the new tin.
Give your answer correct to one decimal place.



7 The diagram shows a sphere and a solid cylinder.
The sphere has radius 8 cm.
The solid cylinder has a base radius of 4 cm and a height of h cm.
The total surface area of the cylinder is half the total surface area of the sphere.
Work out the ratio of the volume of the sphere to the volume of the cylinder.
Give your answer in its simplest form.



Answers:

Numbers

1.

a) 11 b) 2

c) 36 d) 125

2.

a) $\frac{1}{125}$ b) $\frac{4}{9}$

c) 5 *d.* $\frac{9}{4}$, *e.* 0 *f.* 1

3.

a) $7^{\frac{1}{2}}$ b) $2^{\frac{1}{3}}$

c) $5^{\frac{1}{3}}$ d) $3^{\frac{2}{3}}$

e) $\frac{1}{2^{\frac{2}{3}}}$ or $2^{-\frac{2}{3}}$

4.

a	$15\sqrt{2}$	**b**	$\sqrt{5}$
c	$3\sqrt{2}$	**d**	$\sqrt{3}$
e	$6\sqrt{7}$	**f**	$5\sqrt{3}$

5.

3 **a** -1 **b** $9-\sqrt{3}$

c $10\sqrt{5}-7$ **d** $26-4\sqrt{2}$

6.

a $\frac{3+\sqrt{5}}{4}$ **b** $\frac{2(4-\sqrt{3})}{13}$ **c** $\frac{6(5+\sqrt{2})}{23}$

7.

$x-y$

8.

a $3+2\sqrt{2}$ **b** $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

Algebra:

1.

a	$\frac{3x^3}{2}$	**b**	$5x^2$
c	$3x$	**d**	$\frac{y}{2x^2}$
e	$y^{\frac{1}{2}}$	**f**	c^{-3}
g	$2x^6$	**h**	x

2.

a $x^3 + x^{-2}$ **b** $x^3 + x$ **c** $x^{-2} + x^{-7}$

3. *a.* $27x^{\frac{22}{15}}$; *b.* $-\frac{1}{128x^{\frac{13}{2}}}$; 'c. $-\frac{32}{x^{12}}$; *d.* $\frac{1}{12}x^{\frac{11}{5}}$;

4. a) $(x+6y)(x-2y)$ b) $(-x+7)(x+5)$ c) $(3x+1)(2x+1)$ d) $3x(2x-3)(2x+1)$
5. $(x+y)(3b-4)(3b+4)$
6. $(x-4)^2 - y^2 = (x-4-y)(x-4+y)$
7. a) $3(5x-4y)(5x+4y)$ b) $(b-5-a)(b-5+a)$ c) $4x^2y(x-5)^2$

8. $5x^{\frac{1}{3}}(2x^{\frac{1}{24}} - 3 + 4x^{\frac{1}{2}})$

9.

$x^{\frac{3}{2}} - x^{\frac{1}{2}}$ Ans: $x^{\frac{1}{2}}(x-1)$

$4x^{\frac{-2}{3}} - 8x^{\frac{1}{3}}$ Ans: $4x^{-\frac{2}{3}}(1-2x) = \frac{4(1-2x)}{x^{2/3}}$

$(x-5)^{-\frac{1}{2}} - (x-5)^{-\frac{3}{2}}$ Ans: $(x-6)(x-5)^{-\frac{3}{2}} = \frac{x-6}{(x-5)^{3/2}}$

$12x^{\frac{-3}{4}} - 8x^{\frac{1}{4}}$ Ans: $4x^{-3/4}(3-2x) = \frac{4(3-2x)}{x^{3/4}}$

$5(4x+3)^{-1} - 4(5x+1)(4x+3)^{-2}$ Ans: $\frac{11}{(4x+3)^2}$

$-\frac{1}{2}(3x)(1-x^2)^{-\frac{3}{2}}(-2x) + 3(1-x^2)^{-\frac{1}{2}}$ Ans: $\frac{3}{(1-x^2)^{3/2}}$

10.

a $x^2 - 1 - \frac{2}{x^2}$ **b** $x^2 + 2 + \frac{1}{x^2}$

11 $e = \frac{1}{x+7}$ **12** $x = \frac{4y-3}{2+y}$

13 a $r = \sqrt{\frac{A}{\pi}}$ **b** $r = \sqrt[3]{\frac{3V}{4\pi}}$

c $r = \frac{P}{\pi+2}$ **d** $r = \sqrt{\frac{3V}{2\pi h}}$

14 a $x = \frac{abz}{cdy}$ **b** $x = \frac{3dz}{4\pi cpy^2}$

15 $\sin B = \frac{b\sin A}{a}$

16 $\cos B = \frac{a^2+c^2-b^2}{2ac}$

17 a $x = \frac{q+pt}{q-ps}$ **b** $x = \frac{3py+2pqy}{3p-apq} = \frac{y(3+2q)}{3-aq}$

Quadratics:

1.

a $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$ **b** $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

2.

$x^2 + 6x + 9 = 0$

$x^2 + 9x + 20 = 0$

$2x^2 - 10x + 8 = 0$

$x^2 + 5x + 10 = 0$

$x^2 + 6x + 3 = 0$

$2x^2 + 6x + 4 = 0$

$3x^2 - 5x = -4$

$9x^2 - 6x = -9$

$10x^2 - 4x = 8$

. $3x^2 - 2x - 5 = 0$

3 $\frac{1}{2} < x < 3$

4 $x < -\frac{3}{2}$ or $x > \frac{1}{2}$

5 $-3 \le x \le 4$

6 $-3 \le x \le 2$

7 $2 < x < 2\frac{1}{2}$

8 $x \le -\frac{3}{2}$ or $x \ge \frac{5}{3}$

9. **a** $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$ **b** $x = \frac{-3 + \sqrt{23}}{2}$ or $x = \frac{-3 - \sqrt{23}}{2}$

c $x = \frac{5 + \sqrt{13}}{2}$ or $x = \frac{5 - \sqrt{13}}{2}$

Quadratic simultaneous equations:

1 $x = 1, y = 3$

$x = -\frac{9}{5}, y = -\frac{13}{5}$

2 $x = 2, y = 4$

$x = 4, y = 2$

3 $x = 1, y = -2$

$x = 2, y = -1$

4 $x = 4, y = 1$

$x = \frac{16}{5}, y = \frac{13}{5}$

5 $x = 3, y = 4$

$x = 2, y = 1$

6 $x = 7, y = 2$

$x = -1, y = -6$

7 $x = 0, y = 5$

$x = -5, y = 0$

8 $x = -\frac{8}{3}, y = -\frac{19}{3}$

$x = 3, y = 5$

9 $x = -2, y = -4$

$x = 2, y = 4$

10 $x = \frac{5}{2}, y = 6$

$x = 3, y = 5$

11 $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$

$x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$

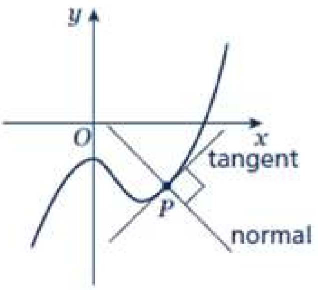
12 $x = \frac{-1+\sqrt{7}}{2}, y = \frac{3+\sqrt{7}}{2}$

$x = \frac{-1-\sqrt{7}}{2}, y = \frac{3-\sqrt{7}}{2}$

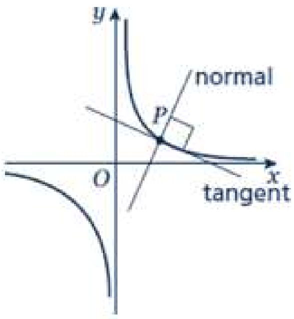
Graphs and transformations:

1 **a** i – C
ii – E
iii – B
iv – A
v – F
vi – D

b **ii**



iv



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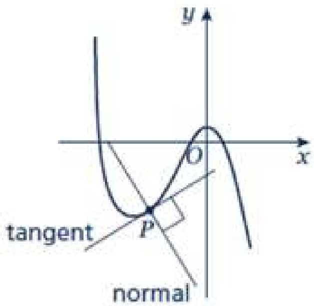

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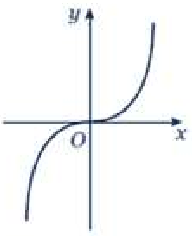

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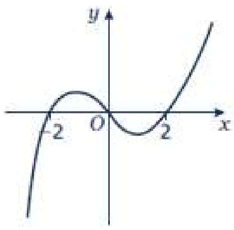

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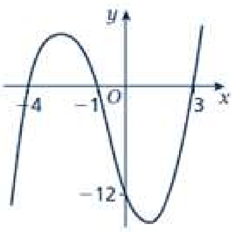

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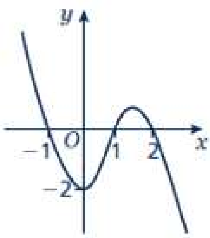

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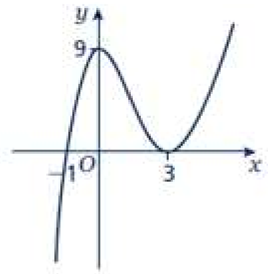

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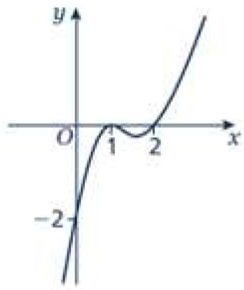

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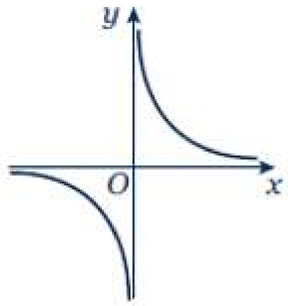

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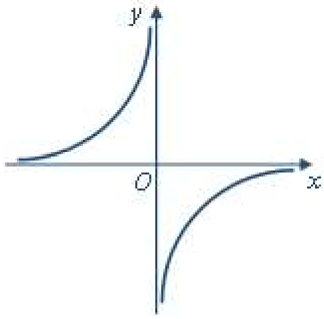

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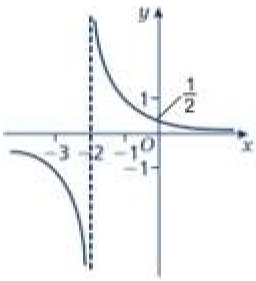

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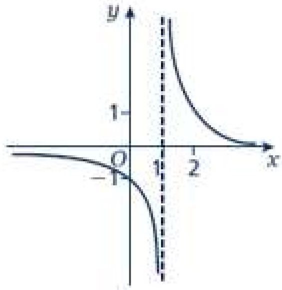

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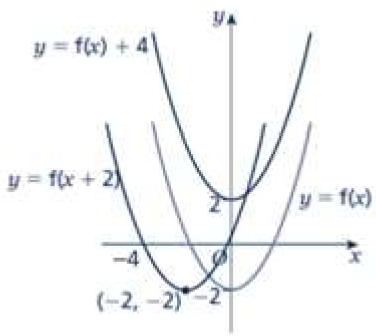

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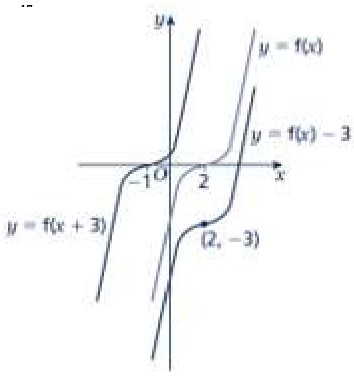
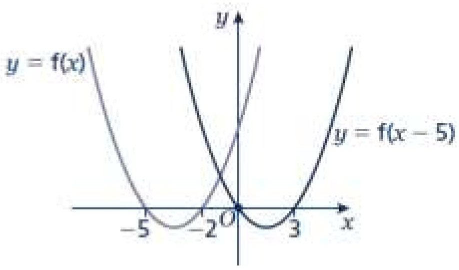


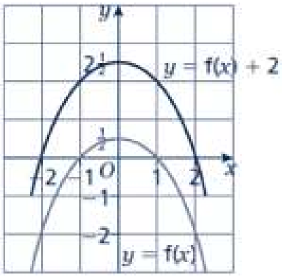
14



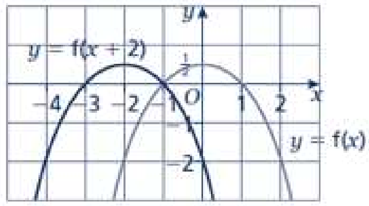
15 C_1: $y = f(x - 90°)$
C_2: $y = f(x) - 2$

16 C_1: $y = f(x - 5)$
C_2: $y = f(x) - 3$

17 **a**



b



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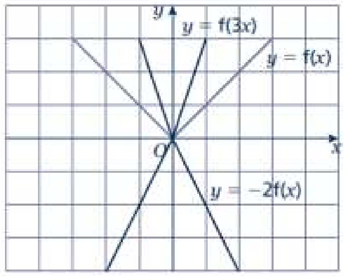

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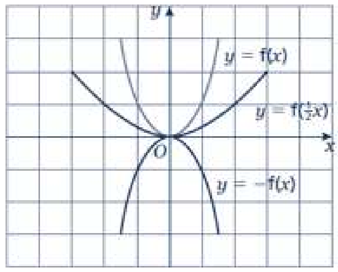

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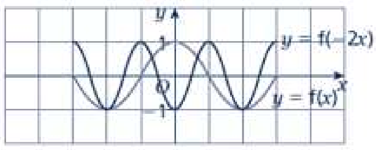
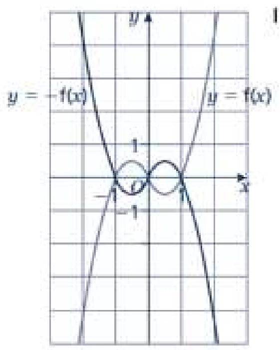
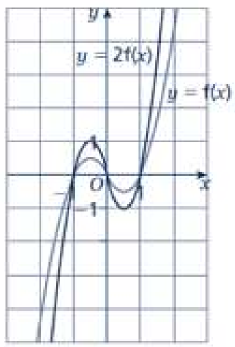


21 $y = -2f(2x)$ or $y = 2f(-2x)$

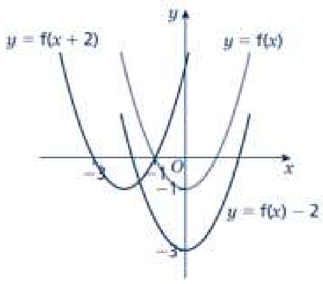
22 **a**



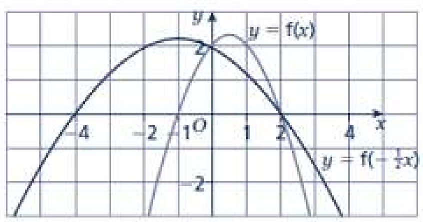
b



23



24



Area under graph:

1 34 units^2

2 149 units^2

3 14 units^2

4 $25\frac{1}{4}$ units^2

5 $6\frac{1}{4}$ units^2

Volumes and Surface Areas:

1 a $V = 396$ cm^3 b $V = 75\,000$ cm^3

c $V = 402.5$ cm^3 d $V = 200\pi$ cm^3

e $V = 1008\pi$ cm^3 f $V = \frac{1372}{3}\pi$ cm^3

g $V = 121.5\pi$ cm^3 h $V = 18\pi$ cm^3

i $V = 48\pi$ cm^3 j $V = \frac{98}{3}\pi$ cm^3

2 17 cm

3 17 cm

4 $V = x^3 + \frac{17}{2}x^2 + 4x$

5 60 cm^3

6 21.4 cm

7 32 : 9